Контрольная работа по теме «Интегралы»
(*Сборник задач по высшей математике. Минорский В.П.*)

1264.

1) $\int\left(x^2+2x+\frac{1}{x}\right)dx=\frac{x^3}{3}+x^2+\ln|x|+C$

2) $\int\frac{10x^8+3}{x^4}dx=\int\left(10x^4+3x^{-4}\right)dx=2x^5+\frac{3x^{-3}}{-3}+C=2x^5-\frac{1}{x^3}+C$

1271.

1) $\int\sin^2\frac{x}{2}dx=\int\frac{1-\cos x}{2}dx=\frac{1}{2}\int(1-\cos x)dx=\frac{1}{2}(x-\sin x)+C$

2) $\int\cos^2\frac{x}{2}dx=\int\frac{1+\cos x}{2}dx=\frac{1}{2}\int(1+\cos x)dx=\frac{1}{2}(x+\sin x)+C$

1273.

1)

$$\int\frac{(x^2-1)^2}{x^3}dx=\int\frac{x^4-2x^2+1}{x^3}dx=\int\left(x-\frac{2}{x}+\frac{1}{x^3}\right)dx=\frac{x^2}{2}-2\ln|x|+\frac{x^{-2}}{-2}+C=$$

$$=\frac{x^2}{2}-2\ln|x|-\frac{1}{2x^2}+C$$

2)

$$\int\left(\frac{1}{\sqrt[3]{x^2}}-\frac{1}{x\sqrt{x}}\right)dx=\int\left(\frac{1}{\sqrt[3]{x^2}}-\frac{1}{\sqrt{x^3}}\right)dx=\int\left(x^{-\frac{2}{3}}-x^{-\frac{3}{2}}\right)dx=\frac{x^{-\frac{2}{3}+1}}{-\frac{2}{3}+1}-\frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1}+C=$$

$$=\frac{x^{\frac{1}{3}}}{\frac{1}{3}}-\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}+C=3x^{\frac{1}{3}}+\frac{2}{x^{\frac{1}{2}}}+C=3\sqrt[3]{x}+\frac{2}{\sqrt{x}}+C$$

1284.

$$\int\sqrt{4x-1}\,dx=\begin{bmatrix}4x-1=t\\ dt=4dx\end{bmatrix}=\frac{1}{4}\int\sqrt{t}\,dt=\frac{1}{4}\cdot\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{4}\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{\sqrt{t^3}}{6}+C=\frac{\sqrt{(4x-1)^3}}{6}+C$$

1285.

$$\int (3-2x)^4 dx = -\frac{1}{2}\int (3-2x)^4 d(3-2x) = -\frac{1}{2}\cdot\frac{(3-2x)^5}{5} + C = -\frac{(3-2x)^5}{10} + C$$

1294.

$$\int tg\,x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{d(\cos x)}{\cos x} = -\ln|\cos x| + C$$

1298.

$$\int \frac{dx}{x(1+\ln x)} = \int \frac{d(1+\ln x)}{1+\ln x} = \ln|1+\ln x| + C$$

1306.

$$\int e^{x^3} x^2\,dx = \begin{bmatrix} x^3 = t \\ dt = 3x^2 dx \end{bmatrix} = \frac{1}{3}\int e^t\,dt = \frac{1}{3}e^t + C = \frac{1}{3}e^{x^3} + C$$

1307.

$$\int e^{-x^2} x\,dx = \begin{bmatrix} -x^2 = t \\ dt = -2x\,dx \end{bmatrix} = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t + C = -\frac{1}{2}e^{-x^2} + C$$

1314.

$$\int \frac{\sqrt{1+\ln x}\,dx}{x} = \begin{bmatrix} 1+\ln x = t \\ dx = \dfrac{dx}{x} \end{bmatrix} = \int \sqrt{t}dt = \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}\sqrt{t^3} + C = \frac{2}{3}\sqrt{(1+\ln x)^3} + C$$

1317.

$$\int \left(e^x + e^{-x}\right)^2 dx = \int \left(e^{2x} + 2e^x e^{-x} + e^{-2x}\right)dx = \int \left(e^{2x} + 2 + e^{-2x}\right)dx = \frac{1}{2}e^{2x} + 2x - \frac{1}{2}e^{-2x} + C =$$

$$= \frac{1}{2}(e^{2x} - e^{-2x}) + 2x + C$$

1324.

$$\int \frac{1-2\sin x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx - 2\int \frac{\sin x}{\cos^2 x}dx = \int \frac{1}{\cos^2 x}dx + 2\int \frac{d(\cos x)}{\cos^2 x} = tgx - \frac{2}{\cos x} + C$$

1325.

$$\int \frac{1+\sin 2x}{\sin^2 x}dx = \int \frac{1}{\sin^2 x}dx + \int \frac{\sin 2x}{\sin^2 x}dx = \int \frac{1}{\sin^2 x}dx + \int \frac{2\sin x\cos x}{\sin^2 x}dx =$$

$$= \int \frac{1}{\sin^2 x}dx + 2\int \frac{d(\sin x)}{\sin x} = -ctg\,x + 2\ln|\sin x| + C$$

1330.

1) $\int \frac{dx}{x^2-25} = \int \frac{dx}{x^2-5^2} = \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$

2) $\int \frac{dx}{x^2+9} = \int \frac{dx}{x^2+3^2} = \frac{1}{3} arctg \frac{x}{3} + C$

1337.

1)

$$\int \frac{x+1}{\sqrt{x^2+1}}dx = \int \frac{x}{\sqrt{x^2+1}}dx + \int \frac{1}{\sqrt{x^2+1}}dx = \int \frac{d(x^2+1)}{2\sqrt{x^2+1}} + \int \frac{1}{\sqrt{x^2+1}}dx =$$
$$= \sqrt{x^2+1} + \ln\left|x + \sqrt{x^2+1}\right| + C$$

2)

$$\int \frac{x+1}{\sqrt{1-x^2}}dx = \int \frac{x}{\sqrt{1-x^2}}dx + \int \frac{1}{\sqrt{1-x^2}}dx = -\int \frac{d(1-x^2)}{2\sqrt{1-x^2}} + \int \frac{1}{\sqrt{1-x^2}}dx =$$
$$= -\sqrt{1-x^2} + \arcsin x + C$$

1342.

$$\int \frac{dx}{\sqrt{x^2+2x+3}} = \int \frac{dx}{\sqrt{x^2+2x+1+2}} = \int \frac{d(x+1)}{\sqrt{(x+1)^2+\left(\sqrt{2}\right)^2}} = \ln\left|x+1+\sqrt{x^2+2x+3}\right| + C$$

1351.

$$\int \frac{x^2dx}{x^2-2} = \int \frac{x^2-2+2}{x^2-2}dx = \int \frac{x^2-2}{x^2-2}dx + \int \frac{2}{x^2-2}dx = \int dx + 2\int \frac{1}{x^2-\left(\sqrt{2}\right)^2}dx =$$
$$= x + 2\cdot\frac{1}{2\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C = x + \frac{1}{\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right| + C$$

1352.

$$\int \frac{x^4dx}{x^2+2} = \int \frac{x^4-4+4}{x^2+2}dx = \int \frac{(x^2)^2-2^2}{x^2+2}dx + \int \frac{4}{x^2+2}dx = \int \frac{(x^2-2)(x^2+2)}{x^2+2}dx + 4\int \frac{1}{x^2+2}dx =$$
$$= \int (x^2-2)dx + 4\int \frac{1}{x^2+\left(\sqrt{2}\right)^2}dx = \frac{x^3}{3} - 2x + \frac{4}{\sqrt{2}} arctg \frac{x}{\sqrt{2}} + C$$

1359.

$$\int \frac{dx}{\sqrt{4x^2+4x+3}} = \int \frac{dx}{\sqrt{4(x^2+x)+3}} = \frac{1}{2}\int \frac{dx}{\sqrt{x^2+x+\frac{3}{4}}} = \frac{1}{2}\int \frac{dx}{\sqrt{x^2+2\cdot\frac{1}{2}x+\left(\frac{1}{2}\right)^2+\frac{1}{2}}} =$$

$$= \frac{1}{2}\int \frac{d\left(x+\frac{1}{2}\right)}{\sqrt{\left(x+\frac{1}{2}\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}} = \frac{1}{2}\ln\left|x+\sqrt{x^2+x+\frac{3}{4}}\right|+C$$

1363.

$$\int x\ arctg\ x\ dx = \begin{bmatrix} \int udv = uv - \int vdu \\ u = arctg\ x;\ du = \frac{1}{1+x^2}dx \\ dv = x\,dx;\ v = \frac{x^2}{2} \end{bmatrix} = \frac{x^2}{2}arctg\ x - \frac{1}{2}\int \frac{x^2}{1+x^2}dx =$$

$$= \frac{x^2}{2}arctg\ x - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}dx = \frac{x^2}{2}arctg\ x - \frac{1}{2}\left(\int dx - \int \frac{dx}{1+x^2}\right) =$$

$$= \frac{x^2}{2}arctg\ x - \frac{1}{2}(x - arctg\ x) + C = \frac{x^2+1}{2}arctg\ x - \frac{x}{2} + C$$

1364.

$$\int x^2\cos x\ dx = \begin{bmatrix} u = x^2;\ du = 2xdx \\ dv = \cos x\,dx;\ v = \sin x \end{bmatrix} = x^2\sin x - 2\int x\sin x\ dx = \begin{bmatrix} u = x;\ du = dx \\ dv = \sin x\,dx;\ v = -\cos x \end{bmatrix} =$$

$$= x^2\sin x - 2\left(-x\cos x + \int \cos x\ dx\right) = x^2\sin x - 2(-x\cos x + \sin x) + C =$$

$$= x^2\sin x + 2x\cos x - 2\sin x + C$$

1372.

$$\int x^3e^{-x}dx = \begin{bmatrix} u = x^3;\ du = 3x^2dx \\ dv = e^{-x}dx;\ v = -e^{-x} \end{bmatrix} = -x^3e^{-x} + 3\int x^2e^{-x}dx = \begin{bmatrix} u = x^2;\ du = 2xdx \\ dv = e^{-x}dx;\ v = -e^{-x} \end{bmatrix} =$$

$$= -x^3e^{-x} + 3\left(-x^2e^{-x} + 2\int xe^{-x}dx\right) = \begin{bmatrix} u = x;\ du = dx \\ dv = e^{-x}dx;\ v = -e^{-x} \end{bmatrix} =$$

$$= -x^3e^{-x} + 3\left(-x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}dx\right)\right) = -x^3e^{-x} + 3\left(-x^2e^{-x} + 2\left(-xe^{-x} - e^{-x}\right)\right) + C =$$

$$= -x^3e^{-x} + 3\left(-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right) + C = -x^3e^{-x} - 3x^2e^{-x} - 6xe^{-x} - 6e^{-x} + C$$

1375.

$$\int\sqrt{x}\ln x dx=\begin{bmatrix} u=\ln x;\ du=\dfrac{dx}{x} \\ dv=\sqrt{x}\,dx;\ v=\dfrac{x^{3/2}}{3/2}=\dfrac{2}{3}x\sqrt{x} \end{bmatrix}=\frac{2}{3}x\sqrt{x}\ln x-\frac{2}{3}\int\sqrt{x}\,dx=$$

$$=\frac{2}{3}x\sqrt{x}\ln x-\frac{2}{3}\cdot\frac{2}{3}x\sqrt{x}+C=\frac{2}{3}x\sqrt{x}\ln x-\frac{4}{9}x\sqrt{x}+C$$

1383.

$$\int\sin^2 3x dx=\int\frac{1-\cos 6x}{2}dx=\int\left(\frac{1}{2}-\frac{\cos 6x}{2}\right)dx=\frac{x}{2}-\frac{1}{12}\sin 6x+C$$

1390.

$$\int\sin^5 x dx=\int\sin^4 x\cdot\sin x dx=-\int(1-\cos^2 x)^2 d(\cos x)=[\cos x=t]=-\int(1-t^2)^2 dt=$$

$$=-\int(1-2t^2+t^4)dt=-\left(t-\frac{2}{3}t^3+\frac{1}{5}t^5\right)+C=-t+\frac{2}{3}t^3-\frac{1}{5}t^5+C=-\cos x+\frac{2}{3}\cos^3 x-\frac{1}{5}\cos^5 x+C$$

1391.

$$\int\sin^2 x\cos^3 x dx=\int\sin^2 x(1-\sin^2 x)\cos x\,dx=\begin{bmatrix}\sin x=t\\ dt=\cos x dx\end{bmatrix}=\int t^2(1-t^2)\,dt=\int(t^2-t^4)\,dt=$$

$$=\frac{t^3}{3}-\frac{t^5}{5}+C=\frac{\sin^3 x}{3}-\frac{\sin^5 x}{5}+C$$

1397.

$$\int\frac{dx}{\sin 2x}=\int\frac{\sin^2 x+\cos^2 x}{2\sin x\cos x}dx=\int\left(\frac{\sin^2 x}{2\sin x\cos x}+\frac{\cos^2 x}{2\sin x\cos x}\right)dx=\int\left(\frac{\sin x}{2\cos x}+\frac{\cos x}{2\sin x}\right)dx=$$

$$=\frac{1}{2}\int\frac{\sin x}{\cos x}dx+\frac{1}{2}\int\frac{\cos x}{\sin x}dx=-\frac{1}{2}\int\frac{d(\cos x)}{\cos x}+\frac{1}{2}\int\frac{d(\sin x)}{\sin x}=-\frac{1}{2}\ln|\cos x|+\frac{1}{2}\ln|\sin x|+C=$$

$$=\frac{1}{2}(\ln|\sin x|-\ln|\cos x|)+C=\frac{1}{2}\ln\left|\frac{\sin x}{\cos x}\right|+C=\frac{1}{2}\ln|tgx|+C$$

1409.

$$\int (1+3\cos 2x)^2 dx = \int (1+6\cos 2x + 9\cos^2 2x)dx = \int dx + 6\int \cos 2x dx + 9\int \cos^2 2x dx =$$

$$= \int dx + 3\int \cos 2x\, d(2x) + 9\int \frac{1+\cos 4x}{2} dx = \int dx + 3\int \cos 2x\, d(2x) + \frac{9}{2}\int dx + \frac{9}{8}\int \cos 4x\, d(4x) =$$

$$= x + 3\sin 2x + \frac{9}{2} + \frac{9}{8}\sin 4x + C = \frac{11}{2}x + 3\sin 2x + \frac{9}{8}\sin 4x + C$$

1419.

$$\int \frac{x^3}{x-2} dx$$

Выделим целую часть дроби. Разделим числитель на знаменатель.

$$\begin{array}{r|l} x^3 & x-2 \\ \underline{x^3-2x^2} & \overline{x^2+2x+4} \\ 2x^2 & \\ \underline{2x^2-4x} & \\ 4x & \\ \underline{4x-8} & \\ 8 & \end{array}$$

$$\int \frac{x^3}{x-2} dx = \int \left(x^2 + 2x + 4 + \frac{8}{x-2} \right) dx = \frac{x^3}{3} + x^2 + 4x + 8\ln|x-2| + C$$

1426.

$$\int \frac{2x^2-5x+1}{x^3-2x^2+x} dx$$

Представим дробь в виде суммы элементарных дробей.

$$\frac{2x^2-5x+1}{x^3-2x^2+x} = \frac{2x^2-5x+1}{x(x^2-2x+1)} = \frac{2x^2-5x+1}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + Bx(x-1) + Cx}{x(x-1)^2} =$$

$$= \frac{Ax^2 - 2Ax + A + Bx^2 - Bx + Cx}{x(x-1)^2} = \frac{x^2(A+B) + x(-2A-B+C) + A}{x(x-1)^2}$$

Приравняем коэффициенты при одинаковых степенях x.

$$\begin{cases} A+B=2 \\ -2A-B+C=-5 \\ A=1 \end{cases} \quad \begin{cases} 1+B=2 \\ -2-B+C=-5 \\ A=1 \end{cases} \quad \begin{cases} B=1 \\ -1+C=-3 \\ A=1 \end{cases} \quad \begin{cases} B=1 \\ C=-2 \\ A=1 \end{cases}$$

Получим дробь. Сделаем проверку вычислений.

$$\frac{2x^2-5x+1}{x^3-2x^2+x}=\frac{1}{x}+\frac{1}{x-1}-\frac{2}{(x-1)^2}=\frac{(x-1)^2+x(x-1)-2x}{x(x-1)^2}=$$
$$=\frac{x^2-2x+1+x^2-x-2x}{x(x-1)^2}=\frac{2x^2-5x+1}{x(x-1)^2}$$

Вычислим интеграл.

$$\int\frac{2x^2-5x+1}{x^3-2x^2+x}dx=\int\left(\frac{1}{x}+\frac{1}{x-1}-\frac{2}{(x-1)^2}\right)dx=\int\frac{1}{x}dx+\int\frac{1}{x-1}d(x-1)-2\int(x-1)^{-2}d(x-1)=$$
$$=\ln|x|+\ln|x-1|-2\frac{(x-1)^{-2+1}}{-2+1}+C=\ln|x(x-1)|+\frac{2}{x-1}+C$$

1427.

$$\int\frac{5x-1}{x^3-3x-2}dx$$

Разложим дробь на сумму элементарных дробей.

$$\frac{5x-1}{x^3-3x-2}=\frac{5x-1}{x^3-x-2x-2}=\frac{5x-1}{x(x^2-1)-2(x+1)}=\frac{5x-1}{x(x-1)(x+1)-2(x+1)}=$$
$$=\frac{5x-1}{(x+1)(x(x-1)-2)}=\frac{5x-1}{(x+1)(x^2-x-2)}=\frac{5x-1}{(x+1)(x+1)(x-2)}=\frac{5x-1}{(x+1)^2(x-2)}=$$
$$=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-2}=\frac{A(x+1)(x-2)+B(x-2)+C(x+1)^2}{(x+1)^2(x-2)}=$$
$$=\frac{A(x^2-x-2)+B(x-2)+C(x^2+2x+1)}{(x+1)^2(x-2)}=\frac{Ax^2-Ax-2A+Bx-2B+Cx^2+2Cx+C}{(x+1)^2(x-2)}=$$
$$=\frac{x^2(A+C)+x(-A+B+2C)-2A-2B+C}{(x+1)^2(x-2)}$$

Приравняем коэффициенты при одинаковых степенях x.

$$\begin{cases}A+C=0\\-A+B+2C=5\\-2A-2B+C=-1\end{cases}\quad\begin{cases}C=-A\\-A+B-2A=5\\-2A-2B-A=-1\end{cases}\quad\begin{cases}C=-A\\B=5+3A\\-3A-10-6A=-1\end{cases}\quad\begin{cases}C=-A\\B=5+3A\\-9A=9\end{cases}$$

$$\begin{cases}C=1\\B=5-3=2\\A=-1\end{cases}$$

$$\frac{5x-1}{x^3-3x-2}=-\frac{1}{x+1}+\frac{2}{(x+1)^2}+\frac{1}{x-2}$$

Вычислим интеграл.

$$\int \frac{5x-1}{x^3-3x-2}dx = \int\left(-\frac{1}{x+1}+\frac{2}{(x+1)^2}+\frac{1}{x-2}\right)dx = -\int\frac{1}{x+1}d(x+1)+2\int\frac{1}{(x+1)^2}d(x+1)+$$

$$+\int\frac{1}{x-2}d(x-2) = -\ln|x+1| - \frac{2}{x+1} + \ln|x-2| + C$$

1445.

$$\int \frac{3x+2}{2x^2+x-3}dx$$

Разложим дробь на сумму элементарных дробей.

$$x = \frac{-1\pm\sqrt{1+24}}{4} = \frac{-1\pm 5}{4}$$

$$x_1 = \frac{-1-5}{4} = \frac{-6}{4} = -\frac{3}{2}$$

$$x_2 = \frac{-1+5}{4} = 1$$

$$\frac{3x+2}{2x^2+x-3} = \frac{3x+2}{2(x-1)\left(x+\frac{3}{2}\right)} = \frac{3x+2}{(x-1)(2x+3)} = \frac{A}{x-1}+\frac{B}{2x+3} = \frac{A(2x+3)+B(x-1)}{(x-1)(2x+3)} =$$

$$= \frac{2Ax+3A+Bx-B}{(x-1)(2x+3)} = \frac{x(2A+B)+3A-B}{(x-1)(2x+3)}$$

Приравняем коэффициенты при одинаковых степенях x.

$$\begin{cases}2A+B=3\\3A-B=2\end{cases}\qquad\begin{cases}2A+B=3\\5A=5\end{cases}\qquad\begin{cases}B=1\\A=1\end{cases}$$

$$\frac{3x+2}{2x^2+x-3} = \frac{1}{x-1}+\frac{1}{2x+3}$$

Найдем интеграл.

$$\int\frac{3x+2}{2x^2+x-3}dx = \int\frac{1}{x-1}dx + \int\frac{1}{2x+3}dx = \int\frac{1}{x-1}d(x-1) + \frac{1}{2}\int\frac{1}{x+\frac{3}{2}}d(x+\frac{3}{2}) =$$

$$= \ln|x-1| + \frac{1}{2}\ln\left|x+\frac{3}{2}\right| + C$$

1454.

$$\int \frac{dx}{x^2+5x} = \int \frac{dx}{x^2+2\cdot\frac{5}{2}x+\left(\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2} = \int \frac{d\left(x+\frac{5}{2}\right)}{\left(x+\frac{5}{2}\right)^2-\left(\frac{5}{2}\right)^2} = \frac{1}{5}\ln\left|\frac{x+\frac{5}{2}-\frac{5}{2}}{x+\frac{5}{2}+\frac{5}{2}}\right|+C = \frac{1}{5}\ln\left|\frac{x}{x+5}\right|+C$$

1458.

$$\int \frac{x+1}{\sqrt[3]{3x+1}}dx = \begin{bmatrix} \sqrt[3]{3x+1}=t \\ 3x+1=t^3 \\ x=\frac{t^3-1}{3} \\ dx=t^2dt \end{bmatrix} = \int \frac{\frac{t^3-1}{3}+1}{t}\cdot t^2dt = \int \frac{(t^3-1+3)t}{3}dt = \frac{1}{3}\int (t^4+2t)dt = \frac{1}{3}\left(\frac{t^5}{5}+t^2\right)+C =$$

$$= \frac{t^2}{15}\left(t^3+5\right)+C = \frac{1}{15}\sqrt[3]{(3x+1)^2}\left(3x+1+5\right)+C = \frac{1}{15}\sqrt[3]{(3x+1)^2}\left(3x+6\right)+C =$$

$$= \frac{1}{5}\sqrt[3]{(3x+1)^2}\left(x+2\right)+C$$

1484.

$$\int \frac{\sqrt{x}dx}{\sqrt{x}+1} = \begin{bmatrix} \sqrt{x}+1=t \\ \sqrt{x}=t-1 \\ dt=\frac{dx}{2\sqrt{x}} \\ dx=2\sqrt{x}dt=2(t-1)dt \end{bmatrix} = \int \frac{(t-1)\cdot 2(t-1)dt}{t} = 2\int \frac{(t-1)^2dt}{t} = 2\int \frac{(t^2-2t+1)dt}{t} =$$

$$= 2\int\left(t-2+\frac{1}{t}\right)dx = 2\left(\frac{t^2}{2}-2t+\ln|t|\right)+C = t^2-4t+2\ln|t|+C =$$

$$= (\sqrt{x}+1)^2-4(\sqrt{x}+1)+2\ln\left|\sqrt{x}+1\right|+C = x+2\sqrt{x}+1-4\sqrt{x}-4+2\ln\left|\sqrt{x}+1\right|+C =$$

$$= x-2\sqrt{x}-3+2\ln\left|\sqrt{x}+1\right|+C$$

1494.

$$\int\sqrt{4x+x^2}\;dx=\int\sqrt{x^2+4x+4-4}dx=\int\sqrt{(x+2)^2-4}\;dx=\begin{bmatrix}x+2=t\\ dt=dx\end{bmatrix}=\int\sqrt{t^2-4}\,dt=$$

$$=\begin{bmatrix}t=\dfrac{2}{\cos u}\\ dt=\dfrac{2\sin u}{\cos^2 u}du\end{bmatrix}=\int\sqrt{\frac{4}{\cos^2 u}-4}\cdot\frac{2\sin u}{\cos^2 u}du=\int\sqrt{\frac{4-4\cos^2 u}{\cos^2 u}}\cdot\frac{2\sin u}{\cos^2 u}du=\int\sqrt{\frac{4\sin^2 u}{\cos^2 u}}\cdot\frac{2\sin u}{\cos^2 u}du=$$

$$=\int\frac{2\sin u}{\cos u}\cdot\frac{2\sin u}{\cos^2 u}du=4\int\frac{\sin^2 u}{\cos^3 u}du=4\int\frac{(1-\cos^2 u)}{\cos^3 u}du=4\int\frac{1}{\cos^3 u}du-4\int\frac{1}{\cos u}du=$$

$$=\begin{bmatrix}\int xdy=xy-\int ydx\\ x=\dfrac{1}{\cos u};\ dx=\dfrac{\sin u}{\cos^2 u}du\\ dy=\dfrac{du}{\cos u};\ y=tg\,u\end{bmatrix}=4\left(\frac{tg\,u}{\cos u}-\int tg\,u\frac{\sin u}{\cos^2 u}du\right)-4\int\frac{1}{\cos u}du=$$

$$=\frac{4tg\,u}{\cos u}-4\int\frac{tg^2 u}{\cos u}du-4\int\frac{1}{\cos u}du$$

Вычисляем интеграл (перебрасываем интеграл через знак «=»).

$$I=\frac{2tg\,u}{\cos u}-2\int\frac{1}{\cos u}du=2\sec u\cdot tg\,u-2\int\sec u\,du=2\sec u\cdot tg\,u-2\int\frac{\sec^2 u+\sec u\cdot tg\,u}{\sec u+tg\,u}du=$$

$$=\begin{bmatrix}z=\sec u+tg\,u\\ dz=\sec u\cdot tg\,u\,du\end{bmatrix}=2\sec u\cdot tg\,u-2\int\frac{1}{z}dz=2\sec u\cdot tg\,u-2\ln z+C=$$

$$=2\sec u\cdot tg\,u-2\ln\left|\sec u+tg\,u\right|+C=2\sqrt{y^2+1}\cdot y-2\ln\left|\sqrt{y^2+1}+y\right|+C=$$

$$=2\sec u\cdot tg\,u-2\ln\left|\sqrt{tg^2u+1}+tg\,u\right|+C=\frac{1}{2}t\sqrt{t^2-4}-2\ln\left|\frac{t}{2}+\frac{1}{2}\sqrt{t^2-4}\right|+C=$$

$$=\sqrt{4x-x^2}+\frac{1}{2}\sqrt{4x+x^2}-2\ln\left|1+\frac{x}{2}+\frac{1}{2}\sqrt{4x+x^2}\right|+C$$

1535.

$$\int\frac{\sqrt{1+x}dx}{x}=\begin{bmatrix}\sqrt{1+x}=t\\ 1+x=t^2\\ x=t^2-1\\ dx=2tdt\end{bmatrix}=\int\frac{2t^2dt}{t^2-1}=2\int\frac{t^2-1+1}{t^2-1}dt=2\int\left(1+\frac{1}{t^2-1}\right)dt=2\int\left(1+\frac{1}{(t-1)(t+1)}\right)dt$$

$$\frac{1}{(t-1)(t+1)}=\frac{A}{t-1}+\frac{B}{t+1}=\frac{At+A+Bt-B}{(t-1)(t+1)}=\frac{t(A+B)+A-B}{(t-1)(t+1)}$$

$$\begin{cases} A+B=0 \\ A-B=1 \end{cases} \quad \begin{cases} B=-A \\ 2A=1 \end{cases} \quad \begin{cases} B=-\dfrac{1}{2} \\ A=\dfrac{1}{2} \end{cases}$$

$$\frac{1}{(t-1)(t+1)}=\frac{1}{2(t-1)}-\frac{1}{2(t+1)}$$

$$I=2\int\left(1+\frac{1}{(t-1)(t+1)}\right)dt=2\int\left(1+\frac{1}{2(t-1)}-\frac{1}{2(t+1)}\right)dt=2\left(t+\frac{1}{2}\ln|t-1|-\frac{1}{2}\ln|t+1|\right)+C=$$

$$=2t+\ln|t-1|-\ln|t+1|+C=2\sqrt{1+x}+\ln\left|\sqrt{1+x}-1\right|-\ln\left|\sqrt{1+x}+1\right|+C$$

1545.

$$\int x\,tg^2x\,dx=\int x\left(\frac{\sin^2 x}{\cos^2 x}\right)dx=\int x\left(\frac{1-\cos^2 x}{\cos^2 x}\right)dx=\int x\left(\frac{1}{\cos^2 x}-1\right)dx=\int\left(\frac{x}{\cos^2 x}-x\right)dx=$$

$$=\begin{bmatrix} \int u\,dv=uv-\int v\,du \\ u=x;\ du=dx \\ dv=\dfrac{dx}{\cos^2 x};\ v=tg\,x \end{bmatrix}=x\,tgx-\int tg\,x\,dx-\int x\,dx=x\,tgx-\int\frac{\sin x}{\cos x}dx-\int x\,dx=$$

$$=x\,tgx-\ln|\cos x|-\frac{x^2}{2}+C$$

1555.

$$\int\frac{dx}{\left(1+\sqrt{x}\right)^3}=\begin{bmatrix} \sqrt{x}=t \\ x=t^2 \\ dx=2tdt \end{bmatrix}=\int\frac{2tdt}{(1+t)^3}$$

Разложим дробь на сумму элементарных дробей.

$$\frac{2t}{(1+t)^3}=\frac{A}{1+t}+\frac{B}{(1+t)^2}+\frac{C}{(1+t)^3}=\frac{A(1+t)^2+B(1+t)+C}{(1+t)^3}=\frac{A+2At+At^2+B+Bt+C}{(1+t)^3}=$$

$$=\frac{At^2+t(2A+B)+A+B+C}{(1+t)^3}$$

Приравняем коэффициенты при одинаковых степенях *t*.

$$\begin{cases} A=0 \\ 2A+B=2 \\ A+B+C=0 \end{cases} \quad \begin{cases} A=0 \\ B=2 \\ 2+C=0 \end{cases} \quad \begin{cases} A=0 \\ B=2 \\ C=-2 \end{cases}$$

Получим дробь.

$$\frac{2t}{(1+t)^3}=\frac{2}{(1+t)^2}-\frac{2}{(1+t)^3}$$

$$I=\int\frac{2tdt}{(1+t)^3}=\int\left(\frac{2}{(1+t)^2}-\frac{2}{(1+t)^3}\right)dt=\frac{2(1+t)^{-2+1}}{-2+1}-\frac{2(1+t)^{-3+1}}{-3+1}+C=-\frac{2}{1+t}+\frac{1}{(1+t)^2}+C=$$
$$=-\frac{2}{1+\sqrt{x}}+\frac{1}{\left(1+\sqrt{x}\right)^2}+C$$

1566.

$$\int\frac{dx}{1+tgx}=\left[\begin{aligned}&tg\,x=t\\&x=arctg\,t\\&dx=\frac{dt}{1+t^2}\end{aligned}\right]=\int\frac{dt}{(1+t)(1+t^2)}$$

Разложим дробь на сумму элементарных дробей.

$$\frac{1}{(1+t)(1+t^2)}=\frac{A}{1+t}+\frac{Bt+C}{1+t^2}=\frac{A(1+t^2)+(Bt+C)(1+t)}{(1+t)(1+t^2)}=\frac{A(1+t^2)+(Bt+C)(1+t)}{(1+t)(1+t^2)}=$$
$$=\frac{A+At^2+Bt+Bt^2+C+Ct}{(1+t)(1+t^2)}=\frac{t^2(A+B)+t(B+C)+A+C}{(1+t)(1+t^2)}$$

Приравняем коэффициенты при одинаковых степенях *t*.

$$\begin{cases}A+B=0\\B+C=0\\A+C=1\end{cases}\quad\begin{cases}B=-A\\C=-B=A\\A+A=1\end{cases}\quad\begin{cases}B=-\frac{1}{2}\\C=\frac{1}{2}\\A=\frac{1}{2}\end{cases}$$

$$\frac{1}{(1+t)(1+t^2)}=\frac{1}{2(1+t)}+\frac{-t+1}{2(1+t^2)}=\frac{1}{2(1+t)}-\frac{t-1}{2(1+t^2)}$$

$$I=\int\frac{dt}{(1+t)(1+t^2)}=\int\left(\frac{1}{2(1+t)}-\frac{t-1}{2(1+t^2)}\right)dt=\frac{1}{2}\int\frac{dt}{1+t}-\frac{1}{2}\int\frac{tdt}{1+t^2}+\frac{1}{2}\int\frac{dt}{1+t^2}=$$
$$=\frac{1}{2}\ln|1+t|-\frac{1}{4}\ln\left|1+t^2\right|+\frac{1}{2}arctg\,t+C=\frac{1}{2}\ln|1+tg\,x|-\frac{1}{4}\ln\left|1+tg^2x\right|+\frac{1}{2}x+C$$